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Compressible Flow: Turbulence at the Surface

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In a study of compressible flow, we have tracked the motion of particles that float on a turbulent body of water. The second moment of longitudinal velocity differences scales as in incompressible flow. However the separation $\langle R^2(t)\rangle$ of particle pairs does not vary in time according to the Richardson–Kolmogorov prediction $\langle R^2(t)\rangle \propto t^3$. As expected, the self diffusion $\langle d^2(t)\rangle$ shows a cross-over between ballistic motion $\langle d^2(t)\rangle \propto t^2$ at small t and uncorrelated motion $\langle d^2(t)\rangle \propto t$ in the longtime limit.

KEY WORDS: Turbulence; compressible; Richardson; diffusion; float; surface.

1. INTRODUCTION

New effects can appear in turbulent fluids when the incompressibility constraint $\nabla \cdot \mathbf{v} = 0$ is relaxed. If compressible, the fluid particles can momentarily gather at some points and disappear from others. Moreover, the few exact equations that hold for isotropic turbulence of incompressible fluids, such as the Kolmogorov "4/5 law" can no longer be derived. Likewise, doubt is now cast on the famous Richardson diffusion relation, according to which the mean square separation $\langle R^2(t)\rangle$ of particle pairs varies as

$$\langle R^2(t)\rangle \propto t^3.$$

With incompressibility no longer there to aid in separating particle pairs, they may have a tendency to coagulate or become trapped, at least momentarily. This effect indeed appears in certain models of turbulence and is also seen in computer simulations.[1, 2]

Strongly compressible flows might appear to be experimentally inaccessible except in supersonic flows. But there is a simple experimental situation where compressible flows are easily achieved, namely at the

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surface ($z=0$) of an internally turbulent body of fluid. Small particles of low density that *float* at the fluid-air interface, will sample the horizontal components of the velocity $\mathbf{v}_s = v_x(x, y, 0, t), v_y(x, y, 0, t)$. Unlike the molecules of the bulk fluid, which can momentarily come up to the surface and recede from it, the floaters cannot. The turbulence should not be so strong as to generate surface waves of appreciable amplitude, a condition that limits the maximum Reynolds number that can be reached. Of course, the motion of floaters under conditions of appreciable wave motion is an interesting subject in itself.[3] It should be noted that the compressibility studied here is not due to particles with large inertia.[4–6]

The water molecules form a divergence-free system at all values of z, including $z=0$ and the floaters, which follow the water molecules at the surface, obey the equation:

$$\partial v_x(x, y, 0, t)/\partial x + \partial v_y(x, y, 0, t)/\partial y = -\partial v_z(x, y, 0, t)/\partial z.$$

The left hand side describes the two-dimensional compressibility of the system of floating particles. Clearly, the underlying flow forces the right hand side to be non-zero, making the two-dimensional divergence substantially large. By covering the surface with floating particles of relatively high surface density $\theta(x, y, t)$ and again photographically recording their motion, one can study the properties of this passive scalar in the compressible case.

Herein we summarize recently published experiments on this subject[7] and report new measurements made on freshly cleaned surfaces. It turns out that contaminants can seriously alter the surface flow. In addition, we present an investigation of the turbulent diffusion of single particles on the surface and the relative motion of particle pairs. It should be noted that the motion of the floaters is not governed by the laws of 2D turbulence despite their confinement to a two-dimensional plane. This is because particles at the surface can exchange both kinetic energy and vorticity with those in the bulk fluid below. In principle then, one does not even have the Kolmogorov paradigm for guidance. It is therefore not clear whether the surface should even adhere to three-dimensional turbulent statistics. For example, there is no longer a dimensional argument to support the well-known approximate result for $D_n(r)$, the nth moment for longitudinal velocity differences on a scale r, namely

$$D_n(r) \equiv \langle((\mathbf{v}(\mathbf{x}+\mathbf{r}) - \mathbf{v}(\mathbf{x})) \cdot (\mathbf{r}/r))^n\rangle \propto (\epsilon r)^{n/3},$$

where ϵ is the mean dissipation rate of the kinetic energy density. Nevertheless the above scaling relation turns out to hold, at least approximately, for $D_2(r)$.

Theoretical predictions and computer simulations indicate that the compressibility of the surface motion magnifies intermittency effects that appear at higher values of n. The influence of strong compressibility on turbulent flow is most easily understood by considering the problem in one dimension.[1] For $D=1$, particles comprising the flow cannot pass each other, making the compressibility $\mathscr{C}$ maximal, where

$$\mathscr{C} \equiv \langle (\nabla \cdot \mathbf{v})^2 \rangle / \langle (\partial v_i / \partial x_j)^2 \rangle.$$

Here the indices denote the spatial coordinates and the brackets indicate an average over the area of vector fields taken from a large number of temporally uncorrelated measurements. Clearly $\mathscr{C}=1$ for $D=1$. For flow in one dimension, the velocity can be described by a potential Φ, where $v(x, t) = \partial \Phi(x, t)/\partial x$. For turbulence, Φ is a function of both space and time. If the temporal fluctuations are sufficiently slow particles in the flow will accumulate, at least temporarily, at points where Φ is a local maximum. It turns out that local trapping can survive even in higher dimensions. Compressible flows have been intensively studied for the Kraichnan model, which is a simplification of true fluid flow obtained by assuming that the velocity field is Gaussian and delta-correlated in time. According to this model, trapping does not appear unless $\mathscr{C}$ exceeds a certain critical value $\mathscr{C}_c$, which depends on the dimensionality d of the system. For this work $d=2$ and $\mathscr{C}_c=0.5$.

2. THE EXPERIMENTS

The measurements were carried out in a square Plexiglas container 1 m in size. The tank was typically filled with filtered water to a height of 30 cm. In the initial experiments the turbulence was created by a large vertically oscillating grid.[7] In the most recent work the turbulence was generated using a large (6 kW) variable speed pump to force water through a planar grid of pipes placed 8 cm above the bottom of the tank and parallel to the surface. Thirty seven twin jets are distributed evenly between the junctions of the grid, and are free to rotate in the plane of the surface, but below it by several centimeters. This complicated scheme was created to maximize the Reynolds number while at the same time minimizing the amplitude of the surface waves. Typically, the rms amplitude of the randomly moving surface waves was of the order of 1 mm.

In the previously published experiments, the rms velocity fluctuations of the surface particles was a third of that measured using particles moving a cm or so below the surface. This large difference, which is not predicted by computer simulations or order-of-magnitude estimates,[7] was tracked down to surface contamination. To increase the surface turbulence to the value just beneath, it is necessary to continuously clean the surface, and to keep the mean spacing of the particles sufficiently large. Surface cleaning was accomplished by vacuuming the surface in the following way. A bottle was placed in one corner of the tank with its mouth just below the waters surface. Suction from the intakes of the pump were used to drain water from the bottle while water from the tank continually entered the container. The rate was adjusted so as to lower the water surface inside the bottle to a constant height, below that of the tank. This technique pulls water primarily from the surface, and is capable of cleaning an initially dirty surface in a matter of minutes. Even air contaminants are sufficient to create a stiff "skin" at the surface that impedes the motion of the floaters. We do not fully understand this inter-particle interaction.

Measurements of $D_2(r)$ were made in the bulk and on the surface. In the bulk we used polystyrene spheres having a diameter of 10 μm. These spheres have a specific gravity slightly larger than that of water, however the turbulent fluctuations were sufficiently large to consider them neutrally buoyant. In both the surface and bulk measurements the velocity field was measured by tracking the particles with a high speed camera running at several hundred Hz. The camera has a resolution of 1024 by 1024 pixels. The imaged particles were larger than a pixel in order to permit sub-pixel measurements of position, and velocity. The spatial resolution depends on image size which was from 9 to 20 cm on a side. The spatial resolution for single point measurements is typically a fraction of a mm. Illumination was furnished by a pulsed Nd-Yag laser or a 5 W solid state laser.

Initially the floating particles, which ranged in diameter from 10 to 200 μm, were mushroom spores, hollow glass spheres, or talc. The particles, who's density was relatively close to that of water, were held on the surface by surface tension. In the new experiments we also used hollow glass spheres having a specific gravity of 1/4 and a mean radius of 45 μm.[8] When introduced below the surface, these light particles quickly float to the top so as to maintain a surface particle density large enough to measure the velocity at all points in the camera's view. This injection scheme eliminates the depletion of particles in the up-welling regions.

At the maximum usable pump speeds, the integral scale of the turbulence l_0 measured at the surface is roughly 3 cm, with the Taylor microscale Reynolds number $\mathrm{Re}_\lambda = v_{\mathrm{rms}}\lambda/\nu$ greater than 100, where ν is the kinematic viscosity of water and $\lambda = \sqrt{\langle v_{\mathrm{rms}}^2\rangle / \langle (dv(x)/dx)^2\rangle}$, where the x-axis is arbitrarily chosen in the horizontal plane. Though the turbulence is roughly isotropic in that plane, all quoted results refer to angular averages.[9]

3. RESULTS

The local clustering of particles moving in compressible flow at the surface is clearly seen in Fig. 1. The image was made as follows: the pump was switched on and a sufficiently large time was allowed to achieve a turbulent steady state. Then the floating particles (which appear white in this photograph) were initially and suddenly spread uniformly on the surface at $t=0$. The image was captured at a time just a fraction of a second later. After a second, the surface is nearly cleared of particles aside from thin densely packed and irregularly-shaped ribbons. After several seconds all particles have been forced to the perimeter of the container where the turbulence is weak. The exposure time was 10 ms and Re_λ was approximately 150. The particles gather near points where the flow is locally downward, forcing the floaters to accumulate in these regions. Because of this clustering effect, the number density $\theta(x, y, t)$ is difficult to measure and has yet to be quantified. The particles in this image are too small to be resolved and differ from those used in the quantitative measurements presented below.

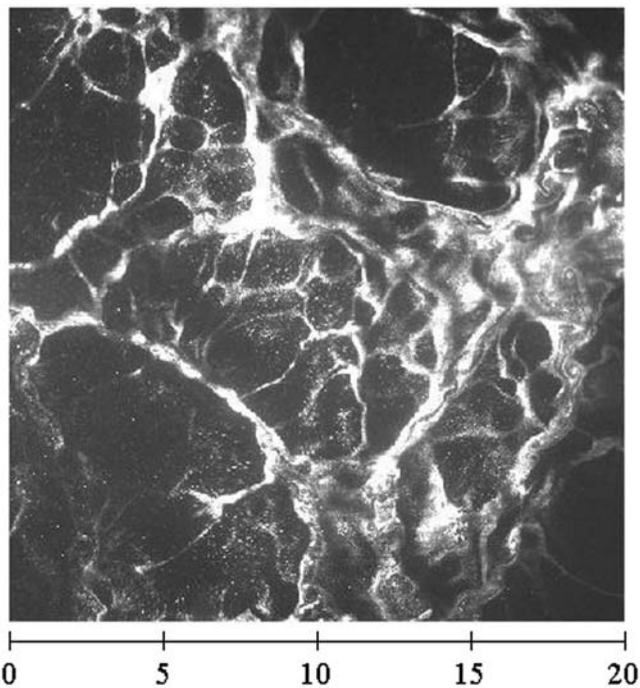

Fig. 1. Image of a cloud of particles on the surface of a turbulent body of water. The particles, which are $\sim 10\ \mu m$ in diameter, were initially dispersed uniformly over the surface. This image was captured 100 ms later. The scale below the figure is in cm.

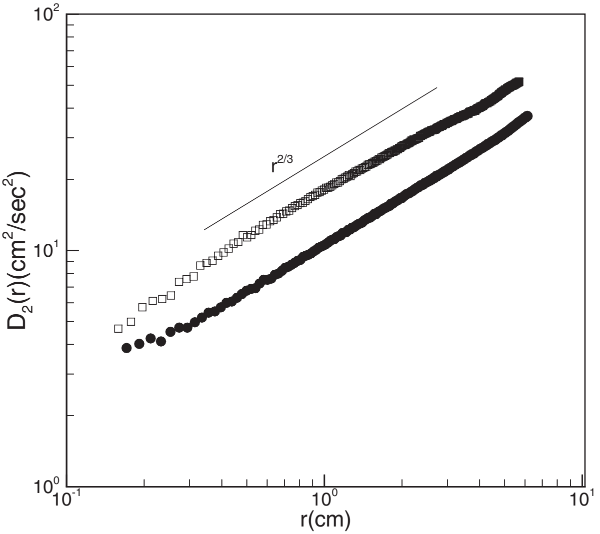

Fig. 2. Second moment of the mean square longitudinal velocity difference as a function of separation r. Upper curve: 1.0 cm below the surface; lower curve: at the surface, $z=0$. The solid line is the Kolmogorov prediction.

Figure 2 is a log-log plot of the second order structure function $D_2(r)$ at the surface, $z=0$, (lower curve) and below the surface at $z=1$ cm (upper curve). The Reynolds numbers Re_λ at $z=0$ and $z=1$ cm were 93 and 120 respectively. At the surface analysis of the velocity fields yield a value for the compressibility of $\mathscr{C}=0.5$, and in the bulk, the Kolmogorov dissipative scale $\eta=(\nu^3/\epsilon)^{1/4}$ is estimated to be 0.1 mm. Here ϵ is found using the following relation. $\epsilon=15\tilde{\eta}\langle(dv(x)/dx)^2\rangle$, where $\tilde{\eta}$ is the viscosity of water (1 cp). From this figure one sees that the data exhibit scaling behavior in the interval $2 \lesssim r \lesssim 60$ mm with a slope very close to $2/3$ (the solid line in the figure), as expected for bulk three-dimensional turbulence. Thus, the strong compressibility and the absence of energy conservation in the inertial range appears to have no effect on this scaling exponent. On the other hand computer simulations of higher moments $D_4(r)$ and $D_6(r)$ at the surface show much stronger intermittency than in the bulk.[2]

The presence of even small waves affect the illumination at the surface. This fluctuating intensity makes it difficult to accurately track individual floating particles for an adequate length of time. Therefore we followed an alternative procedure for these measurements. Larger, (200 μm) hollow glass spheres were initially spread uniformly on the surface and were used to measure the local velocity field. We then followed the motion of virtual particles initially placed on a grid. They were then allowed to follow the

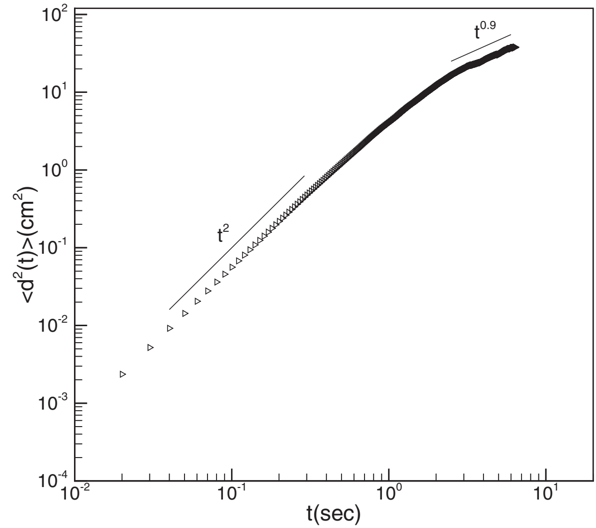

Fig. 3. Single-particle diffusion on the surface. The solid lines have slopes corresponding to ballistic motion (slope = 2) and, at large d, the slope is close to that of the Brownian, which is unity.

measured velocity field, $v_x(x, y, 0, t)$ and $v_y(x, y, 0, t)$. The virtual particles are ideal in that they have no inertia. This scheme introduces some systematic error since the virtual particle velocity is extrapolated from the real velocity field. However the method is very robust to any change in the parameters used to determine the velocity of these particles. Figure 3 is a plot of the single-particle diffusion, $d^2(t) = \langle [x(t) - x(0)]^2 \rangle$. All of the measurements are at $r > \eta$. Very strong projectile motion is seen in the initial interval of ballistic motion $0.4 \lesssim d \lesssim 9$ mm. This scaling persists for a decade beyond the estimated viscous subrange $\eta = 0.1$ mm. The slope decreases smoothly out to $d \simeq 50$ mm, where the motion becomes uncorrelated, that is $d^2(t) \propto t$ (a better fit to the data is a line of slope 0.9 shown in the figure).

For the first time Richardson diffusion was measured in this compressible system. According to the ideas of Richardson and Kolmogorov[10, 11] the mean squared separation of particle pairs, $\langle R^2(t) \rangle = \langle [R(t) - R(0)]^2 \rangle \propto t^a$, in the inertial range $a = 3$. Under the same assumptions $\langle R^2(t) \rangle$ should grow exponentially in the dissipative range. As already noted values of R less than η were inaccessible in this experiment, but it was possible to measure with considerable precision $\langle R^2(t) \rangle$ for over 3 decades in separation. Figure 4 is a plot of $\langle R^2(t) \rangle$, where all particle pairs were initially separated by 3 mm. The thin line of slope 1.65 fits our data rather well for over a decade of R. We lack an understanding of why this slope is roughly

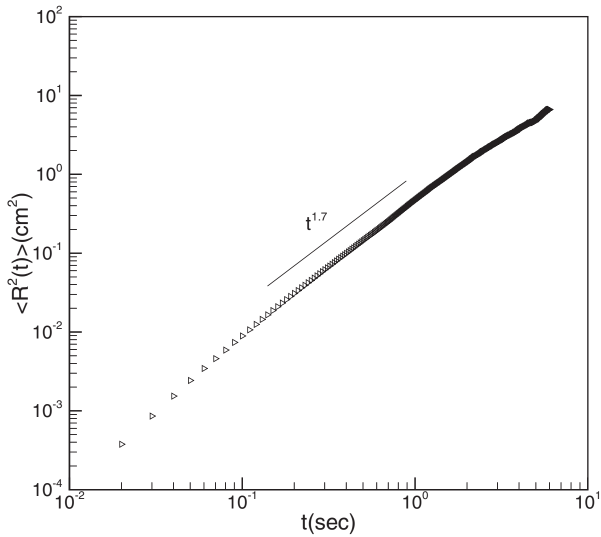

Fig. 4. Growth of the mean square displacement of particles pairs. The initial particle separation is $R = 3$ mm.

half of the expected value of $a = 3$, but it certainly must be connected with the strong compressibility at the surface and particle trapping.

Measurements of $R^2(t)$ were averaged over 27 runs. In each run roughly a thousand virtual particle pairs were tracked for up to 10 s at a data collection rate of 100 Hz. In the individual runs, the slope varied between 1.6 and 1.9, with average being 1.7 and the standard deviation $\sigma = 0.1$. The largest measured pair separation was 20 mm, still within the integral scale $l_0 \simeq 30$ mm, which explains why random Brownian diffusion $\langle R^2(t) \rangle \propto t$ is not seen. Particle trapping and the finite area of measurement prevent tracking particle separations to even larger values of R.

4. SUMMARY

Particles that float on the surface of an incompressible fluid, sample only the horizontal velocity components of the underlying turbulent flow and hence form a strongly compressible system. The upwellings and "downwelling" of the flow produce the spatial localization of the floaters, seen in Fig. 1.

The effect of this compressibility, which is probably best revealed by measuring statistics of the particle surface density, as in Fig. 1, should also be apparent in measurements of the velocity field as revealed in the structure functions $D_n(r)$ and the time variation of particle pair separations

$\langle R^2(t) \rangle$. Our laboratory experiments and the computer simulations of Eckhardt and Schumacher show that $D_2(r)$ scales as in 3D turbulence, but the computer simulations show that higher moments of velocity differences display stronger intermittency than in 3D turbulence. This effect has yet to be verified in the laboratory.

As for Richardson diffusion, we see self-similarity over a wide range separations, but the exponent a in the expression $\langle R^2(t) \rangle \propto t^a$ is much smaller than the Richardson result, $a = 3$, contrary to the computer simulations of Eckhardt and Schumacher. One might expect that large compressibility would diminish a, which is consistent with the experimental result, $a \simeq 1.7$, for measurements where R is in the inertial range (For R much larger than the integral scale of the turbulence, $l_0 \simeq 3$ cm, the relative motion the particles should be random, making $a = 1/2$).

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